T₁ & Magnetization Transfer

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Outline

• T₁ and T₂
• Relevance for MRI
• Measuring T₁
• Magnetization Transfer (MT)
• Measuring MT
• Sources of T₁ contrast: T₁ & MT
Magnetization

Nuclear spins polarize in a magnetic field

Energy: $B_0$
Magnetization

Nuclear spins polarize in a magnetic field

$B_0=0$
Magnetization

Nuclear spins polarize in a magnetic field

Energy:

\[ \frac{N_-}{N_+} = e^{-\hbar \gamma B_0 / k_B T} \approx 1 - \hbar \gamma B_0 / k_B T = 1 - 6.5 \times 10^{-6} B_0 \]
Magnetization

Nuclear spins polarize in a magnetic field, but how fast? Change in polarization requires energy transfer to different species.

Pure water: little energy transfer -> slow (change in) polarization
Magnetization

Time course:
- every spin has certain probability to transition
- P_ for \( \uparrow \) P_ for \( \uparrow \) where P_ slightly higher than P_ (due to \( \Delta E \))
- # spins: \( \uparrow N P_ - \uparrow N P_+ \)
- \( M = N_+ - N_ - \)
- change in \( M = dM = (N P_ - N P_+) \)
- \( dM = 0 \) for equilibrium (\( M_0 \)), \( N_+ / N_- = P_- / P_+ \)
- \( M = M_0 + \Delta, \ N_- = N_0 - \Delta/2, \ N_+ = N_0 + \Delta/2, \)
- \( dM = ((N_0 - \Delta/2)P_- - (N_0 + \Delta/2)P_+) = N_0 P_- - N_0 P_+ - \Delta/2(P_- + P_+) = - \Delta/2(P_- + P_+) \)
- \( dM/dt = -k(M - M_0), \ k = R_1 = 1/T_1 \)
Magnetization
Magnetization

\[
\frac{dM_z}{dt} = -R_1^*(M_z - M_0)
\]

\[
M_z = M_0 - (M_z(0) - M_0)e^{-R_1 t}
\]
Magnetization

\[ \frac{M_z}{M_0} = \frac{M_0 - (M_z(0) - M_0)e^{-R_1t}}{M_0} \]

\[ T_1 = \frac{1}{R_1} \]
$T_1$-Relaxation in MRI

$M_z$ not directly measured, $M$ needs to be transverse:
$T_2$-Relaxation

$M$ rotates around $B_0$
Frequency: $\gamma B_0$
$T_1$ & $MT$

**$T_2$-Relaxation**

$M$ rotates around $B_0$
Frequency: $\gamma B_0$
$B_0$ not the same everywhere:
dispersion
$T_2$-Relaxation

$M_t$ in rotating frame:
$T_2$-Relaxation

$M_t = M_t(0)e^{-R_2t}$
T₂-Relaxation

T₂ is dispersion of M in transverse plain caused by frequency differences, from
- spin-spin interactions (true T₂)
- field inhomogeneity from magnet or local susceptibility (T₂*)

No energy transfer, can be (much) faster then T₁
T₁-Relaxation

MR measurement:

1. RF
2. T₂
3. T₁
$T_1$-Relaxation

Relevance for MR Imaging

$M_t=0$, $M_z=1$  $M_t=1$, $M_z=0$  $M_t=0$, $M_z=0$
T₁ & MT

T₁-Relaxation & MRI

M_z

RF  TR  RF  Time  RF  RF

0  2  4  6  8  10
$T_1$-Relaxation & TR
T₁ & MT

T₁-Relaxation & Flip Angle

M_z

45°

90°
$T_1$ & MT

$T_1$-Relaxation & Flip Angle

- $22.5^\circ$
- $45^\circ$
- $90^\circ$

$M_z$ vs. Time

Graph shows the magnetization $M_z$ over time for different flip angles ($22.5^\circ$, $45^\circ$, $90^\circ$). The $M_z$ values are normalized for easier comparison.
$T_1$-Relaxation & Signal

Flip = 90°
$T_1$-Relaxation & Signal

Flip = 45°
T₁ & MT

T₁-Relaxation & Signal

Flip = 22.5°
$T_1$-Relaxation: Signal Calculation

\[ M_z = M_a \]

\[ M_b = \cos(\alpha)M_a \]

\[ M_a = 1 - (1 - M_b)e^{-TR/T_1} = 1 - (1 - M_b)E_1 \]

Solution: \[ M_a = \frac{(1 - E_1)}{(1 - \cos(\alpha)E_1)} \]

Signal: \[ M_t = \sin(\alpha)M_a \]
$T_1$-Relaxation & Signal

![Graph showing the relationship between Signal and Flip Angle for different TR values](image)

- $TR = T_1$
- $TR = 0.5 T_1$
- $TR = 0.25 T_1$
- $TR = 0.125 T_1$
Inversion

More RF than excitation
Inversion Recovery

\[ M_z \]

\[ T_1 \]

\[ 0.7T_1 \]

\[ T_1 \]
$T_1$ Measurement

$T_1$ can be measured in two ways:
- saturation
- inversion recovery
$T_1$ Measurement

Saturation

RF
T₁ Measurement

Saturation

Mz
$T_1$ Measurement

Saturation

Signal
$T_1$-Relaxation & Signal

![Graph showing signal intensity as a function of flip angle for different repetition times (TR). The graph includes curves for $TR = T_1$, $TR = 0.5 \times T_1$, $TR = 0.25 \times T_1$, and $TR = 0.125 \times T_1$.](image-url)
T₁ Measurement

Inversion Recovery

RF

90°  180° 90°  180°  90°  180°  90°
T₁ Measurement

Inversion Recovery

Mz

RF
T₁ Measurement

Inversion Recovery

Mz, Mt

RF

T₁ & MT
$T_1$ Measurement

Inversion Recovery Hybrid: MPRAGE
$T_1$ Measurement

Inversion Recovery Hybrid: MPRAGE
T_1 Measurement

MPRAGE: T_1 contrast

M_{z}, M_{t}

0 500 1000 1500 2000
-1.0 -0.5 0.0 0.5 1.0
**T₁ Measurement**

MPRAGE: T₁ contrast
Signal depends on $T_1$, but also on:
- $T_2(*)$
- RF (flip angle): Transmit coil, Dielectric effects, Calibration
- Receive sensitivity: Coils, System amplification
- Proton density
T₁ Measurement

Choosing a method

Inversion Recovery: best quantification, slow
Saturation: fast, but mixed with RF and some T₂
MPRAGE: fast and useful contrast, hard to quantify, and potential for spatial blurring.
MPRAGE with second scan (MP2RAGE) can compensate some of the coil contrast etc.
$T_1$ Measurement

Examples: 7T IR with EPI
$T_1$ Measurement

Examples: MPRAGE, MP2RAGE

Curtesy of Pascal Sati, NINDS
T₁ Measurement

Examples: 7T 0.5mm MP2RAGE
$T_1$ Measurement

Examples: 7T, MPRAGE, Gd-injection

Enhancing lesion due to open blood-brain barrier

Curtesy of Pascal Sati, NINDS
$T_1$: Sources

Pure water: very little energy transfer -> slow relaxation

Interaction with other molecules required: in the brain, mostly lipids and protein

Interaction: Magnetization Transfer (= part 2)
MT: $T_2$

Average field
Lipid has more structure
MT: $T_2$

Average field $\neq 0$: short $T_2$
MT: $T_2$

Short $T_2$

$T_2 \ll 1 \text{ ms} : \text{not visible in MRI}$

But: ‘hidden’ magnetization interacts with water
Solids in White Matter
T₁ & MT

MT: T₂

Lipid and Exchange
MT: $T_2$

Lipid and Exchange

Lipid Membrane  Water
Lipid and Exchange

Exchange:
- Magnetic
- Chemical

Lipid Membrane  Water

MT
MT
Lipid and Exchange

Macro Molecular Protons  Water Protons
MT Parameters

\[
\begin{align*}
R_{1MP} & \quad k_{MW} \quad R_{1WP} \\
T_{2MP} & \quad k_{WM} \quad T_{2WP}
\end{align*}
\]

f \quad 1-f

MP \quad WP

T_1 & MT
Magnetization measured relative to some baseline: normalization changes the equations and definition of variables.

Normalization can be for total magnetization, the sum of the water pools (for more complex models), or per pool individually.
Starting point: Bloch equation for $M_z$ for one pool:

$$\frac{d M_{wp}}{dt} = -R_{1wp} (M_{wp} - M_{wp,0})$$

Normalize to $M_{wp,0}$:

$$\frac{d M_{wp}}{dt} = -R_{1wp} (M_{wp} - 1)$$
Add second pool (each pool normalized to one):

\[ \frac{d M_{wp}}{dt} = -R_{1wp} (M_{wp} - 1) \]
\[ \frac{d M_{mp}}{dt} = -R_{1mp} (M_{mp} - 1) \]

multiply by relative sizes:

\[ (1-f) \frac{d M_{wp}}{dt} = -(1-f)R_{1wp} (M_{wp} - 1) \]
\[ f \frac{d M_{mp}}{dt} = -f R_{1mp} (M_{mp} - 1) \]
Add exchange:

\[
(1-f) \frac{d M_{wp}}{dt} = -(1-f)R_{1wp} (M_{wp} - 1) - k M_{wp} + k M_{mp} \\
f \frac{d M_{mp}}{dt} = -f R_{1mp} (M_{mp} - 1) - k M_{mp} + k M_{wp}
\]

k= fraction of spin exchanging compared to total
Alternative form (1), divide by size:

\[
\begin{align*}
\frac{dM_{wp}}{dt} &= -R_{1wp} (M_{wp} - 1) - \frac{k}{(1-f)} M_{wp} + \frac{k}{(1-f)} M_{mp} \\
\frac{dM_{mp}}{dt} &= -R_{1mp} (M_{mp} - 1) - \frac{k}{f} M_{mp} + \frac{k}{f} M_{wp}
\end{align*}
\]

with \( k_{mp} = \frac{k}{f} \) and \( k_{wp} = \frac{k}{(1-f)} \):

\[
\begin{align*}
\frac{dM_{wp}}{dt} &= -R_{1wp} (M_{wp} - 1) - k_{wp} M_{wp} + k_{wp} M_{mp} \\
\frac{dM_{mp}}{dt} &= -R_{1mp} (M_{mp} - 1) - k_{mp} M_{mp} + k_{mp} M_{wp}
\end{align*}
\]
Alternative form (2), normalized to sum of pools, substituting $M'_{wp} = (1-f)M_{wp}$, and $M'_{mp} = f M_{mp}$

\[
\frac{d M'_{wp}}{dt} = -R_{1wp} (M'_{wp} - 1) - k/(1-f) M'_{wp} + k/f M'_{mp}
\]
\[
\frac{d M'_{mp}}{dt} = -R_{1mp} (M'_{mp} - 1) - k/f M'_{mp} + k/(1-f) M'_{wp}
\]

with $k_{mp}=k/f$ and $k_{wp}= k/(1-f)$:

\[
\frac{d M'_{wp}}{dt} = -R_{1wp} (M'_{wp} - 1) - k_{wp} M'_{wp} + k_{mp} M'_{mp}
\]
\[
\frac{d M'_{mp}}{dt} = -R_{1mp} (M'_{mp} - 1) - k_{mp} M'_{mp} + k_{wp} M'_{wp}
\]
MT Equations Summary

\[
(1-f)\frac{dM_{wp}}{dt} = -(1-f)R_{1wp} (M_{wp} - 1) - k M_{wp} + k M_{mp} \\
f \frac{dM_{mp}}{dt} = -f R_{1mp} (M_{mp} - 1) - k M_{mp} + k M_{wp}
\]

\[
\frac{dM_{wp}}{dt} = -R_{1wp} (M_{wp} - 1) - k_{wp} M_{wp} + k_{wp} M_{mp} \\
\frac{dM_{mp}}{dt} = -R_{1mp} (M_{mp} - 1) - k_{mp} M_{mp} + k_{mp} M_{wp}
\]

\[
\frac{dM'_{wp}}{dt} = -R_{1wp} (M'_{wp} - 1) - k_{wp} M'_{wp} + k_{mp} M'_{mp} \\
\frac{dM'_{mp}}{dt} = -R_{1mp} (M'_{mp} - 1) - k_{mp} M'_{mp} + k_{wp} M'_{wp}
\]

\[
k_{mp} = \frac{k}{f}, \quad k_{wp} = \frac{k}{(1-f)}, \quad (1-f)k_{wp} = f k_{mp}
\]
MT
Parameters

\[ R_{1\text{MP}} \quad T_{2\text{MP}} \quad f \]

\[ R_{1\text{WP}} \quad T_{2\text{WP}} \quad 1-f \]

\[ k_{\text{MW}} \]

\[ k_{\text{WM}} \]

\[ k_{\text{mp}} = k/f \]
\[ k_{\text{wp}} = k/(1-f) \]
\[(1-f)k_{\text{wp}} = f k_{\text{mp}}\]
MT Equations

Saturation: \( S = 1 - M_z \)

\[
d S_{WP} / dt = -R_{1wp} S_{WP} - k_{WM} S_{WP} + k_{WM} S_{MP}
\]

\[
d S_{MP} / dt = -R_{1mp} S_{MP} - k_{MW} S_{MP} + k_{MW} S_{WP}
\]

\[
d S / dt = R_x S ; \quad R_x = \text{matrix, solution:}
\]

\[
S_{WP}(t) = a_1 e^{-\lambda_1 t} + a_2 e^{-\lambda_2 t} ; \quad \lambda_{1,2} \text{ eigenvalues of } R_x
\]
MT

Saturation \( S_{WP}(t) = a_1 e^{-\lambda_1 t} + a_2 e^{-\lambda_2 t} \)
$T_1$ Measurement

Saturation

RF
MT Measurement

MT Saturation Equilibrium

MT Saturation in balance with $T_1$
Saturation by off-resonance RF

FT
Saturation by off-resonance RF
$T_1$ & MT

$T_1$ Measurement

Inversion Recovery
MT Measurement

MT Saturation Recovery

T₁ & MT

RF

90°

MT

90°

MT

90°

MT

90°
Pulse Design

Short $T_2$

$T_1$ & MT
Effective flip angle $\alpha \approx \gamma B_1 T_2$

$M_z = \cos(\alpha) = 1 - \alpha^2/2$

$PW/T_2$ times
T_1 & MT

**Pulse Effect**

**Short T_2**

**Intermediate T_2**

**Long T_2**
Pulse Design

Approximate Transitions:

Short $T_2$: $(\gamma B_1)^2 PW/2$

Long $T_2$: $\sin(\alpha)^2 PW/2$
MT Recovery

Normalized difference with reference

T<sub>1</sub> & MT

TI: 7 64 145 256 380 ms
Saturation efficiency
Saturation efficiency
Saturation efficiency
MT and spectral properties MPs in human brain Roger Jiang

• MP spectral properties (the saturation effects on MPs as a function of frequency offset):
  • Calculate pair of amplitudes \((a_1(F), a_2(F))\) for every voxel at each \(F\), using decay rates \((\lambda_1, \lambda_2)\)
  • Calculate \(FS_{MP}(0,F) = \frac{a_1(F)(-\lambda_1+k_{WM}+R_{1,WP})}{k_{WM}} + \frac{a_2(F)(-\lambda_2+k_{WM}+R_{1,WP})}{k_{WM}}\)

 Frequency offset \(F\) (from left to right: -16, -8, -4, -2, -1, -0.5, 0.5, 1, 2, 4, 8, 16 kHz)

• Average in WM ROI’s
MT and spectral properties MPs in human brain Roger Jiang

- A Lorentzian line (the black solid curve) and sum of two Lorentzian lines (the red dashed curve) fitting to $FS_{MP}(0,F)$.

- $\Delta F$ of -727 Hz (-2.42ppm), close to -2.34 ppm at 3 T (Hua et al, MRM 2007) and -2.55 ppm at 4.7 T (Pekar et al, MRM 1996).

- 2-Lorentzian fitting: a component (73%) with $T_2$ of 23 $\mu$s was found, consistent with the results in the study on fixed marmoset brain.
T₁ & MT

Inversion & MT

IR and Exchange

MP

WP
Inversion & MT
IR and Exchange

MP

WP
Inversion & MT

IR and Exchange

In an IR experiment initial saturation of MP depends on RF power
Early part of IR dominated by exchange
Inversion & MT

IR double exponential and RF dependent

\[ \ln(S_r) = \ln(1 - M_{zf}) \]

Adiabatic 7.0ms
Composite 1.2ms
Composite 3.6ms
Composite 6.9ms
Inversion & MT

Calculated $T_1$ as function of TI

High RF               Low RF

Adiabatic    Cmp. 1.2ms  Cmp. 3.6ms
MT

Equations

\[
d S_{WP} / dt = -R_{1WP} S_{WP} - k_{WM} S_{WP} + k_{WM} S_{MP} \\
d S_{MP} / dt = -R_{1MP} S_{MP} - k_{MW} S_{MP} + k_{MW} S_{WP} \\
\]

\[
f k_{MW} = (1-f) k_{WM} \\
S_{WP}(t) = a_1 e^{-\lambda_1 t} + a_2 e^{-\lambda_2 t} \\
\lambda_1 \approx (1-f)R_{1WP} + fR_{1MP}
\]
Inversion & MT

\[ R_{1\text{eff}} = \lambda_1 \approx (1-f)R_{1\text{WP}} + fR_{1\text{MP}} \]
Summary

- Pure water has a very long $T_1$
- Main source of $T_1$ relaxation is semi-solid lipids & other macro molecules through MT between water and MP
- Consequences:
  :: MT and $T_1$ contrast both measure MP
  :: $T_1$ relaxation (at least) bi-exponential
Summary

Reality more complex:
- multiple pools of water (intra-, extra- cellular, myelin)
- multiple kinds of MP, each with $R_1$, $T_2$ etc.

Two pool $T_1$ generally sufficient, fast component more important at higher field
Many compartments

Axonal water

Interstitial water

Myelin water
Many compartments

Free water

Myelin water

Macro-molecular Protons
The End